Lesson 2

Thermomechanical Measurements for Energy Systems (MENR)

Measurements for Mechanical Systems and Production (MMER)

A.Y. 2015-16

Zaccaria (Rino) Del Prete

$$\frac{A}{U} = a$$

The property **A** is called: **«measurand»** the reference property **U** is the **«measurement unit»**
the ratio between **A** and **U** today is always performed by a calibrated instrument !
the measurement **a** expresses the amplitude or intensity of the property **A** we are studying …

Direct measurements: when there is a direct comparison between the **measurand A** and the **unit U**

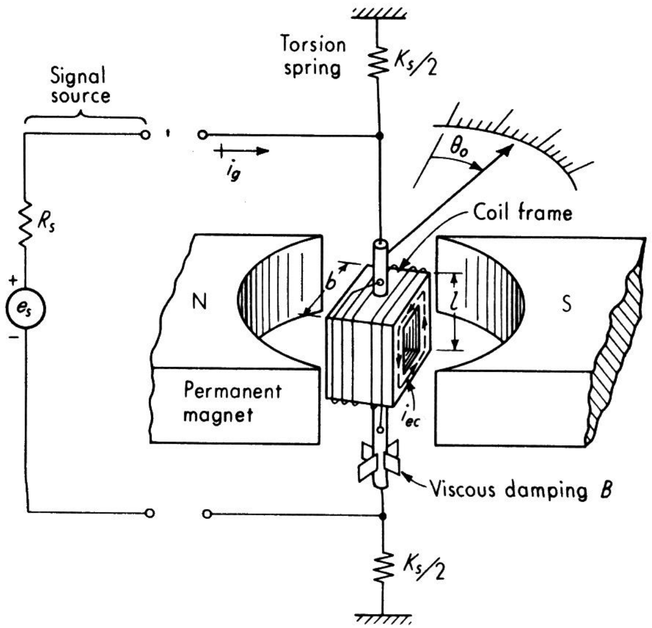
Indirect measurements: when a physical law is applied to measure quantities other than the one we are interested in:
for example, measuring a velocity $v = x/t$ by measuring a distance x and a time t

Measurements with ***calibrated instruments***: by far the majority of the measurements performed today
the instrument has memorized inside the unit U …

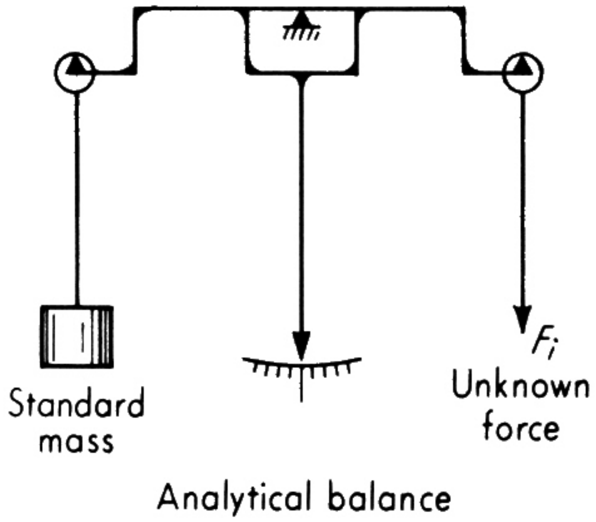
Direct instruments : (only very few)

Indirect instruments : they always transform the physical quantity they acquire in input … → **Analog instruments** / **Digital instruments**

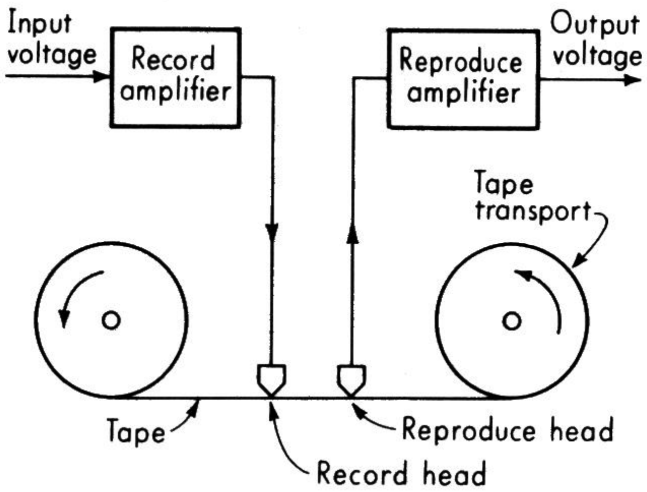
Deflection instruments



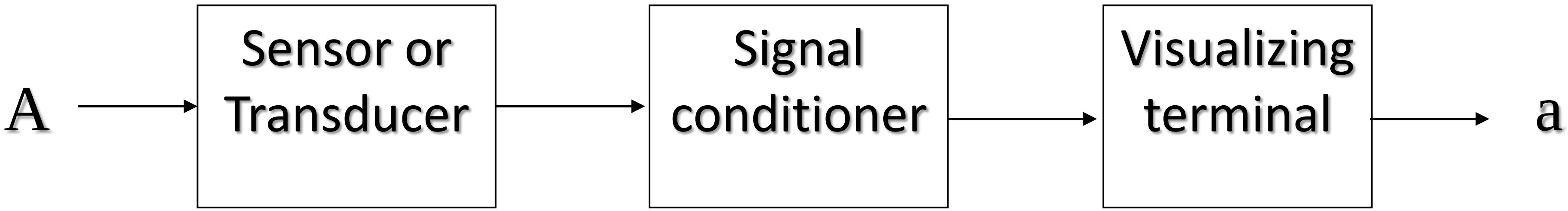
Null-out instruments



Recording instruments



Basic MEASUREMENT CHAIN:



First question to be answered: <u>WHAT</u> do we want to measure and <u>WHY</u> ?

It's a "KNOWLEDGE PATH" that leads to new *quantitative* INFORMATION about the world surrounding us !

- want we to have an approximate control of a physical quantity ?
- want we to do a rigorous scientific measurement ?
- want we to use the value of the measure for an automatic control ?
- want we to know to what extent can a physical parameter vary to set an alarm signal ?

1. we choose the instrument according to the "extension range" of the physical quantity to be measured, based on the *amount of variation* that is expected for the quantity and also on *how fas*t the quantity changes its value during the measurement;
2. we reads the numeric data on the output device of the instrument;
3. by means of the *graduation of the instrument* we associate the numeric data **a** with the **U** units, performing actually the real measurement: **A** = **a × U**
4. at this point we still only have a *raw measurement* of the physical quantity **A**. We have to identify the *many uncertainties* **ε** associated with the raw data, correct the data we obtained and then switch to the final measurement: **A = (a ± ε) × U**
5. with the diffusion of digital instrumentation is now possible to acquire many repeated measurements of a physical quantity a_1 a_2 ... a_n. This raises the question of identifying the *true magnitude* of the value. We have therefore to go through a statistical analysis of the data …

We have to establish the QUALITY of a measurement

High quality measurements means being able to get ***data with low uncertainty***; most of the times, this is also an ***expensive procedure*** !

To quantify the ***quality of a measurement*** we have to quantify first the ***quality of the instrument*** !

To this extent we can define ***5 general metrological characteristics*** that apply to every measuring instrument or measuring system and completely describe its performances !

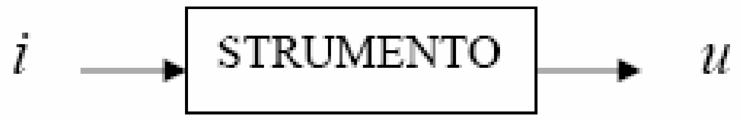
- MEASUREMENT SPAN
- SENSITIVITY
- STIFFNESS
- ACCURACY
- MEASUREMENT RAPIDITY

• MEASUREMENT SPAN

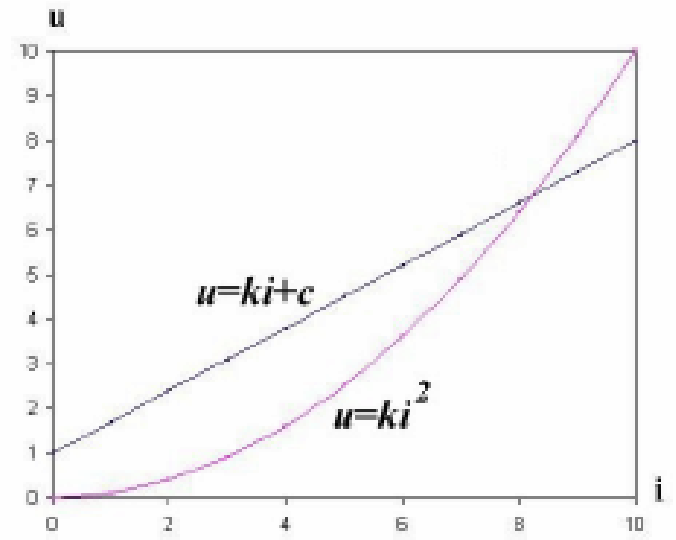
The ***measuring span*** is the numerical *range* between the minimum and maximum values of the "measurand" the instrument can appropriately measure and within which the other four instrument characteristics are valid !

Span = high operation limit – low operation limit

Graduation curve :



$u = f(i)$



The operation of every instrument is based on a *physical law*, this law is described by an equation; the same equation is also *the equation representative of the graduation curve* !

if this equation is of 1st degree (a straight line) the instrument is said *linear instrument* !
if this equation is of 2nd degree (parabolic) the instrument is said *quadratic instrument* !

Example: find the *graduation curve* of the Hg thermometer

the fundamental physical law (starting point) is the law of *volumetric thermal expansion* of all materials (fluids):

$$V = V_0 \left(1 + \alpha \cdot \Delta T\right)$$

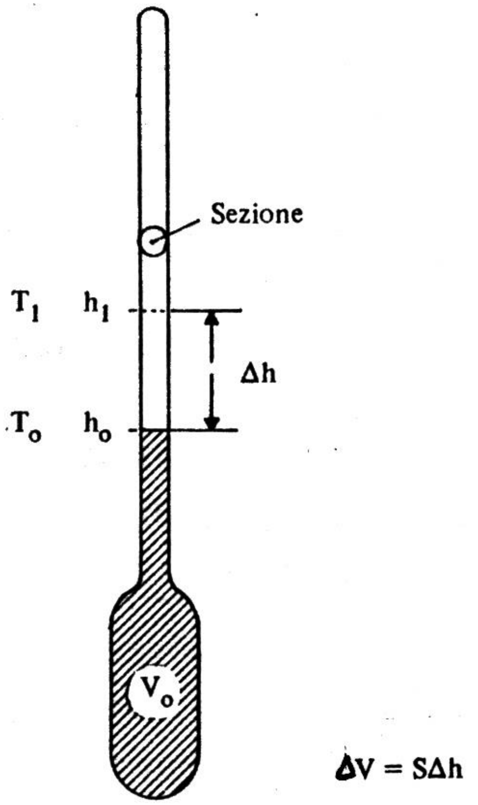
$$V - V_0 = \alpha \cdot V_0 \Delta T \qquad \frac{\Delta V}{V_0} = \alpha \cdot \Delta T$$

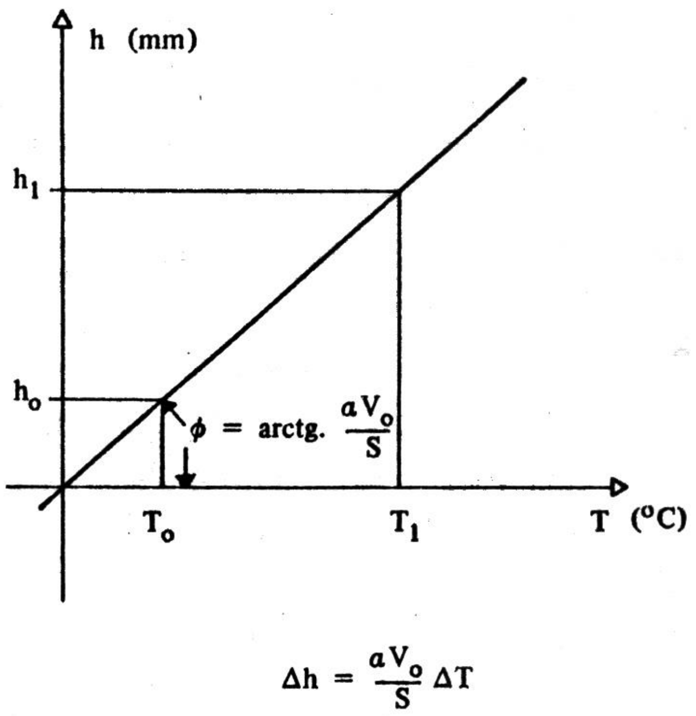
with $\Delta V = S \cdot \Delta h$ $\qquad \dfrac{S \cdot \Delta h}{V_0} = \alpha \cdot \Delta T$

$$\Delta h = \frac{\alpha \cdot V_0}{S} \Delta T$$

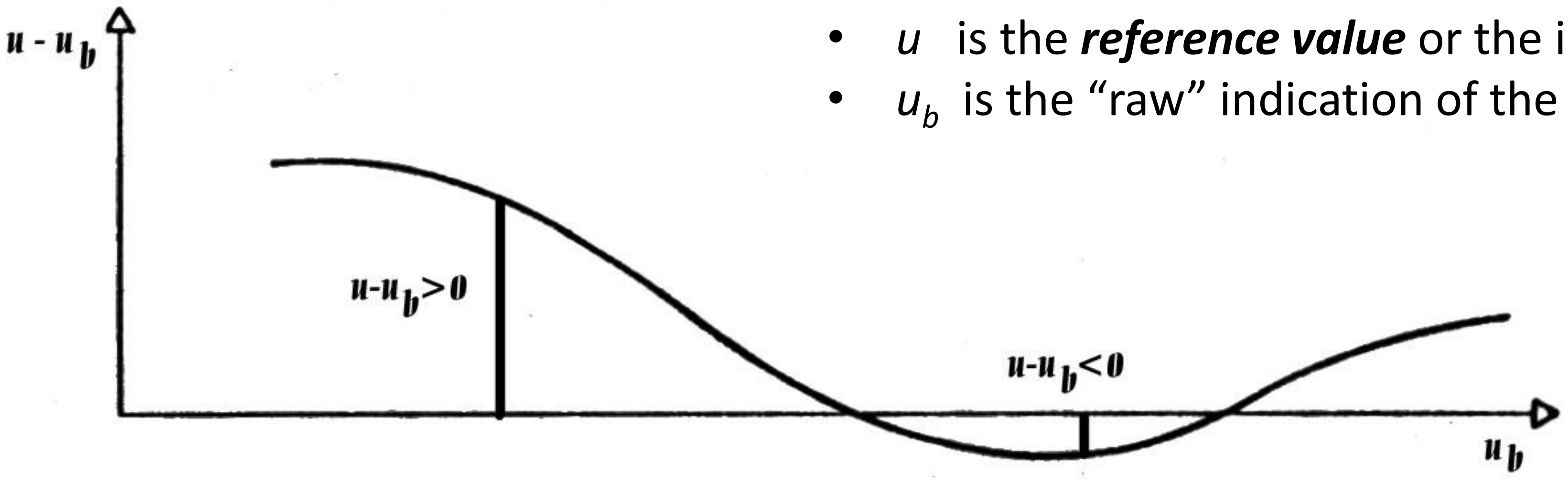
output (↗ Δh) input (measurand) (↖ ΔT)

That’s the ***graduation curve*** !





Graduation curve should NOT be confused with the ***CALIBRATION curve :***



- *u* is the ***reference value*** or the indication of a ***reference instrument***
- u_b is the "raw" indication of the *instrument under calibration*

$$u - u_b = f(u_b)$$

is the ***calibration curve***, ie the curve of the *differences* between the *reference value u* and the *«raw» instrument indication* u_b , for every instrument output u_b .

if $u - u_b < 0$ ⟶ $u < u_b$ the instrument under calibration "overestimates" the measurand magnitude (input) so we have to **subtract** the indicated deviation from u_b .

if $u - u_b > 0$ ⟶ $u > u_b$ the instrument under calibration "underestimates" the measurand magnitude (input) so we have to **add** the indicated deviaton to u_b .

• SENSITIVITY

Capability of the instrument to «sense» ***small variations*** of the input variable (the measurand)

how small can a variation ***Δi*** of the input be, to get from the instrument an *appreciable* output ***Δu*** ?

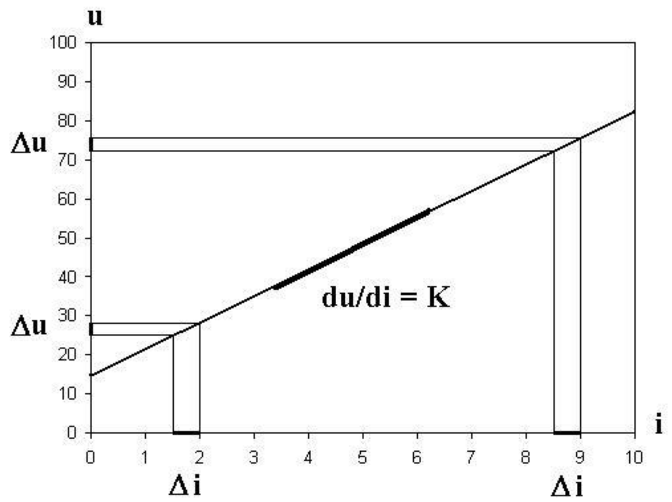
We can certainly write $\frac{\Delta u}{\Delta i}$ that, for variations $\Delta i \to 0$ means $\frac{du}{di}$, or better $\lim_{\Delta i \to 0} \frac{\Delta u}{\Delta i} = \frac{du}{di} = S$

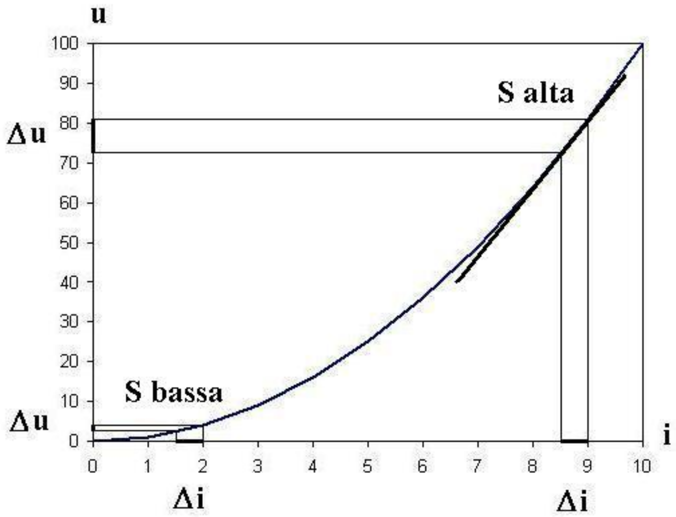
It is immediately observed that, if ***u = f (i)*** is the *graduation curve*, the ***sensitivity*** $S = \frac{du}{di}$ is the ***derivative***

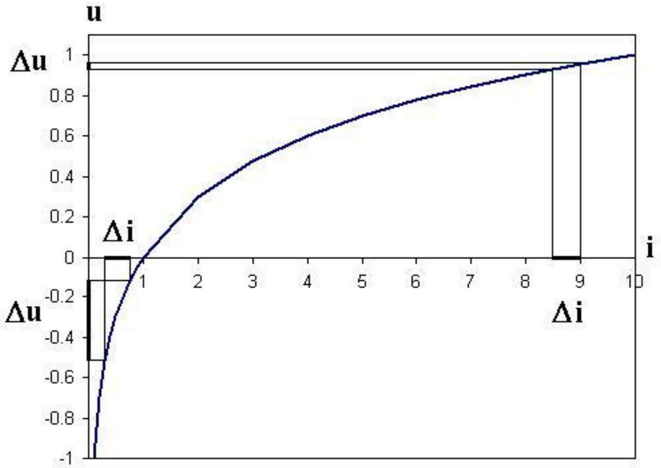
of the graduation curve !

The sensitivity S can be calculated for *every point* of the *graduation curve*, by means of the differential ratio: $\frac{du}{di}$
it has therefore also a geometric meaning:
it's the *slope of the tangent* to the graduation curve in the measuring point that is being considered.

Examples:







Instruments with a
constant sensitivity
are called ***linear instruments*** !

$$u = ki + c \rightarrow S = \frac{du}{di} = K$$

Instruments with a
*sensitivity that is a function of **i***
can be ***quadratic instruments*** !

$$u = ki^2 \rightarrow S = \frac{du}{di} = 2ki$$

Instruments with a
*sensitivity that is a function of **i***
can be ***logarithmic instruments*** !

$$u = k \log i \rightarrow S = \frac{du}{di} = \frac{k}{i}$$

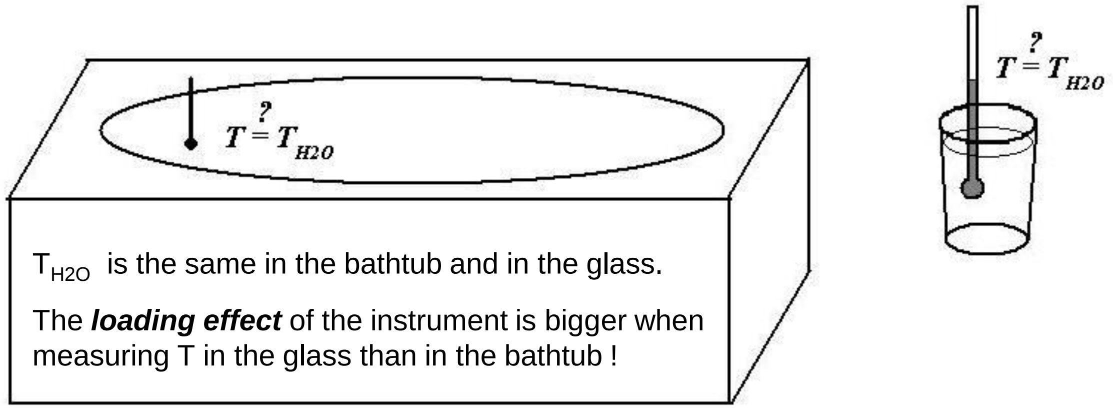
Do not confuse the *sensitivity* with the ***resolution*** of an instrument, which actually is a more appropriate concept for ***digital instruments*** !

• STIFFNESS

The introduction of any *measuring instrument* into a *measured medium* always results in the *extraction of some energy* from the medium, thereby changing the value of the measured quantity from its undisturbed state !

This circumstance makes a *perfect measurement theoretically impossible* !

example:



Every instrument that physically interacts with the measured quantity has a ***«loading effect»*** on the measurand and slightly changes its value

Instrument designer can only *«minimize»* this *loading effect*, which is called ***stiffness*** !

An instrument with a *low stiffness* means an instrument capable of doing measurements with a *small loading effect* !

Stiffness is a "singular characteristic" because it depends also on the *measurement environment* and *circumstances.*

Some *numerical means* of expressing the *loading effect* of the instrument on the measured medium would be helpful in comparing competing instruments at the moment of their choice or purchase !

One parameter could be the ***insertion*** ***or*** ***connection error*** done by the instrument when connecting with the measurand:

$$\varepsilon_{ins} = \frac{a_b - a}{a_b} \cong \frac{a - a_b}{a}$$

where: a_b is the numerical value of the measurand "before" the instrument insertion;
a is the numerical value of the measurand "after" the instrument insertion, and also the *actual measurement* returned by the instrument.

Because a_b is a value that can NOT be measured, this parameter may seem useless !

Let's go to a generalized definition of s***tiffness*** and ***input impedance*** :

At the input of each component in a measuring system there exists a variable q_{i1} , the "***effort variable***", that carries the "*information content" of the measurement*. At the same input, however, there is a second variable q_{i2}, associated with q_{i1}, in a way that the product $q_{i1} \times q_{i2}$ has the "*dimensions of power"* and represents an instantaneous *rate of "energy withdrawal" from the preceding element* !

The *generalized input impedance* can then be defined as : $$Z_{gi} = \frac{q_{i1}}{q_{i2}}$$

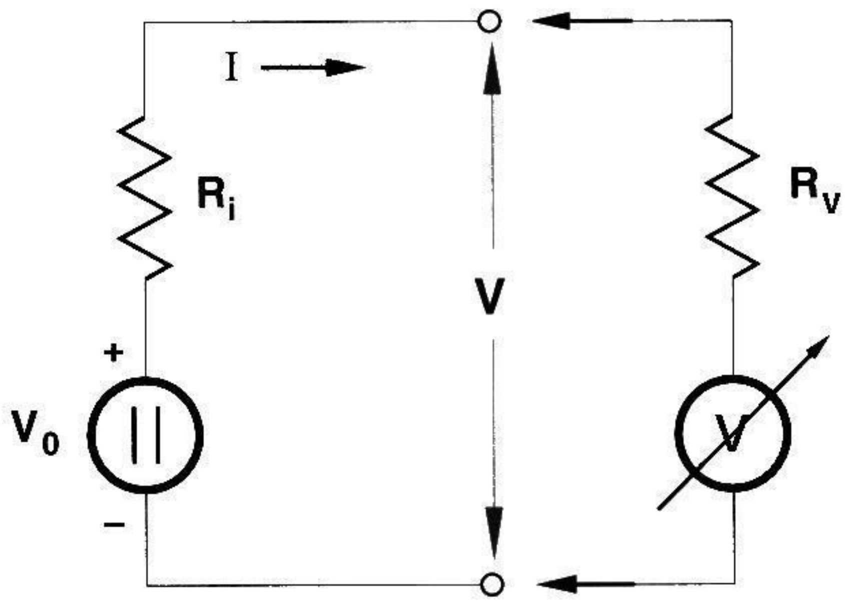
While the *power drained from the preceding element* is : $$P = \frac{q_{i1}^2}{Z_{gi}}$$

and a "*large" generalized input impedance* is needed to keep the *power drain small* !

These concepts can be immediately applied to an electrical example: a *voltmeter* measuring an *unknown voltage V.* As soon as the meter is applied to the voltage terminals, the electrical circuit is changed and the value of *V* is no longer the same but changes to another value V_m .
For a voltmeter the input variable of interest, the *effort variable* (q_{i1}), is the terminal voltage V_m . If we look for an *associated variable* (q_{i2}) which, when multiplied by q_{i1} , gives the power withdrawal from the voltage terminals, we find the meter current i_m meets this requirement.

Thus, the *input impedance* in this example is: $$Z_{gi} = \frac{q_{i1}}{q_{i2}} = \frac{V_m}{i_{m,}} = R_m$$ the ***meter resistance*** !

This situation is very important indeed and applies also to the *internal stages* of a more complex instrument chain, for example, when coupling a transducer that measures a physical quantity *A* and outputs a voltage *V* or a current *i* to the *signal processing stage* that follows in the measurement chain ….



Before connecting the voltmeter to the terminals we have :

$V = V_0$ with $I = 0$

After connecting the voltmeter we have:

$V_0 = (R_i + R_v)\cdot I$ with $I \neq 0$

But the instrument measures:

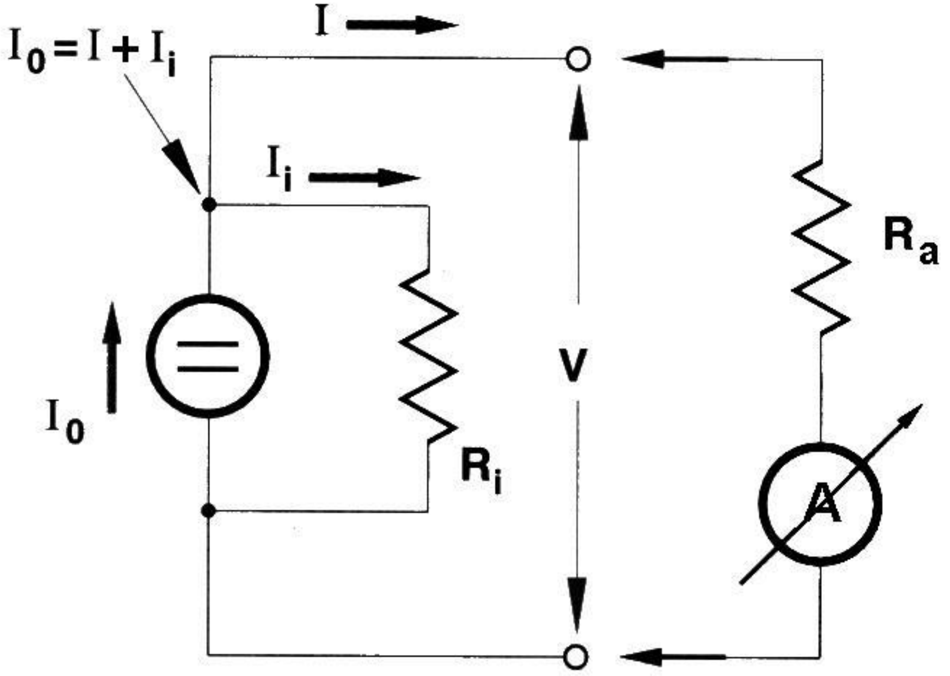
$V = R_v \cdot I \neq V_0$

We commit an error : the "***connection error***" $\varepsilon_{ins} = \dfrac{V_0 - V}{V_0} \Rightarrow \dfrac{(R_i + R_v)\cdot I - R_v \cdot I}{(R_i + R_v)\cdot I} = \dfrac{R_i}{R_i + R_v} = \dfrac{1}{1 + \dfrac{R_v}{R_i}}$

If we wish to keep this error small $\varepsilon_{ins} \to 0$ we have to design either $R_v \to \infty$ or $R_i \to 0$

Acting on the transducer *output resistance* R_i in not so easy therefore, the preferred way in designing the connection is making the *input resistance* R_v of the signal processing stage (voltmeter) very big !

This choice prevents the signal V which carries the "information" about the measurement to degrade further !



Before connecting the ammeter to the terminals we have :

$I_0 = I_i$ with $I = 0$

After connecting the ammeter we have:

$I_0 = I_i + I$ with $I = \frac{V}{R_a}$

And the instrument measures:

$I \neq I_0$

We commit an error : the ***"connection error"*** $\varepsilon_{ins} = \frac{I_0 - I}{I_0} = 1 - \frac{I}{I_0}$ with $V = I_0 \frac{R_i R_a}{R_i + R_a}$ because the two resistances R_i and R_a are now in a parallel configuration $R_i // R_a$

Therefore : $\frac{V}{R_a} = I_0 \frac{R_i}{R_i + R_a}$ $\quad I = I_0 \frac{R_i}{R_i + R_a}$ $\quad \frac{I}{I_0} = \frac{R_i}{R_i + R_a}$

And the connection error becomes : $\varepsilon_{ins} = 1 - \frac{I}{I_0} \Rightarrow 1 - \frac{R_i}{R_i + R_a} = \frac{R_i + R_a - R_i}{R_i + R_a} = \frac{R_a}{R_i + R_a} = \frac{1}{\frac{R_i}{R_a} + 1}$

If we wish to keep this error small $\varepsilon_{ins} \to 0$ we have to design either $R_i \to \infty$ or $R_a \to 0$

Having at disposal a current generator with a big *internal short-circuit resistance* R_i is advisable but not so easy to obtain from a transducer which outputs a current therefore, the preferred way in designing the two stage connection is making the *input resistance* R_a of the signal processing stage (ammeter) very small !

This choice prevents the current signal that carries the “information” on the measurement to degrade further !

So far, all the metrological characteristics we studied are valid for a ***static measurand A***